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A theoretical framework for the compressive properties of aligned fibre composites

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New experimental results have made necessary the reformulation of the theory for compressive strength. The new theory is based on the precept that a number of different mechanisms can cause composite failure. The active one in a particular situation is that which gives the lowest failure stress. Thus composite strength can be dominated by fibre strength when the fibres are ductile, or controlled by matrix yielding when the matrix is soft. Lack of linearity in the fibres has a very important effect also, as does the adhesion between the matrix and the fibres. Modulus is affected as well as strength. Governing equations are developed for six different mechanisms and the agreement with experiment is very good. It is concluded that to make composites with good compressive properties the fibres should be hard, as straight as possible and well bonded to the matrix. The matrix should have a high yield stress, tensile strength and compressive strength. Hybridization is useful to improve the compressive strength of Kevlar composites, but should be avoided with brittle fibre systems due to unfavourable "hybrid effects".

1. Introduction

It is very appropriate at this time to re-examine the theories used to explain the compressive strength of aligned fibre composites. The compressive strength is important in many composite applications. (For example, in flexure, failure is often initiated on the compressive side of the composite). Yet the reasons for the low compressive strength are not understood. The early elastic buckling theory [1], although widely quoted in modern texts, overestimates the compressive strength and fails to account for the observed fibre volume-fraction (V_f) dependence of the strength [2]. A more recent gross matrix yielding theory [3], while giving the correct V_f dependence, requires that the composites fail at the matrix yield strain. This is not the case [4].

Not only does no satisfactory theory exist, but recent work has shown that a number of factors, hitherto not considered, have an important influence on compressive strength. These include fibre strength, matrix yield strength, the adhesion between the fibres and the matrix, and the degree of fibre linearity. In addition, the compressive Young's modulus of aligned fibre composites is often much less than the Rule of Mixtures value.

A new theory is needed which can account for all these factors. Since many possible failure mechanisms must exist, the theory will have many facets and a considerable number of governing equations. The actual failure stress in a given situation will be determined by the failure process which operates at the lowest stress.

This paper will examine the recent data on compressive strength and modulus and will develop governing equations for underlying mechanisms that can account for the majority of the observations. It is hoped that this will spur further activity, the identification of other mechanisms and eventually a more complete understanding of this important and complex process.

2. Fibre strength

The compressive strengths of composites containing ductile fibres are usually dominated by the compressive strengths of the fibres, σ_{fcu}. In the case of aligned steel wire reinforced polymers, Ferran and Harris [5] showed that the composites obeyed a mixture rule expression for composite compressive strength, σ_{1cu} in the fibre direction

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$$\sigma_{1cu} = V_f \sigma_{fcu} + V_m \sigma_{1m}, \tag{1}$$

where V_f and V_m are fibre and matrix volume fractions and the subscript 1 indicates the fibre direction. For σ_m, little error normally results from using the matrix ultimate compressive strength, σ_{mcu}, though with reinforced polymers, where the polymer yield strain is usually greater than the composite failure strain, we should strictly write

$$\sigma_{1m} = \sigma_{fcu} E_m / E_f, \tag{2}$$

where E_m and E_f are the Young's moduli of the matrix and fibre, respectively.

Equation 1 was obeyed for wires which were hard and for wires which were quite soft. In both cases, σ_{fcu} was quite close to the fibre tensile strength.

A more exhaustive series of experiments on this type of composite [6] showed that Equation 1 was obeyed within a few per cent over the range of fibre volume fractions tested (0 to 0.34) and σ_{fcu} correlated well with the flexural strength of the steel wires. Tests were carried out with steels having a wide range of strengths, obtained by varying the heat treatment of one batch of steel. The softer steels failed by plastic collapse, while the composites containing the hardest steels failed explosively when the fibre stress reached about 1.2 GPa. It was concluded from these experiments that σ_{fcu} was equal to the fibre compressive strength. Buckling did not appear to initiate failure, although it occurred after the composite reached its maximum stress. The fibres were extremely straight initially; they were selected so that their radii of curvature were greater than 2200 diameters.

With tungsten fibre reinforced copper, Equation 1 was also obeyed to within a few per cent [2] over the whole range of V_f values tested (0 to 0.75). σ_{fcu} was about 50% higher than the tensile strength of the fibres. (In this case σ_m should strictly be given by the stress the matrix can support at the fibre failure strain. This is beyond the yield strain of the matrix. Again, however, little error results if σ_{mcu} is used for σ_{1m}.)

Kevlar reinforced polymers also obey Equation 1 quite closely [7]. σ_{fcu}, however, is less than one tenth of the fibre tensile strength. In addition, the fibre appears to have a reduced Young's modulus in compression of about 60% of the tensile value. With nylon fibres Equation 1 is also obeyed, but $\sigma_{fcu} \simeq \sigma_{mcu}$ [6].

With glass and carbon fibres we obtain the linear behaviour indicated by Equation 1, but rather than Equation 1 we should use

$$\sigma_{1cu} = V_f \sigma_{fmax} + V_m \sigma_{1m} \tag{3}$$

since there is no evidence to indicate that, with these non-yielding fibres, there is any correlation between the fibre stress at composite failure, σ_{fmax}, and the ultimate compressive strength of the fibres. Glass and stiff carbon have σ_{fmax} values close to the fibre tensile strengths, but the strong carbon fibres fail at a composite stress such that σ_{fmax} is less than half the tensile strength [7]. Equation 3 is obeyed with glass and the carbons for $V_f = 0$ to 0.4. For $V_f > 0.4$ the strength falls below σ_{1cu} in Equation 3, the reduction increasing with increasing V_f.

3. Fibre linearity

Chaplin [8] has shown, with a series of carefully made composites, that fibre linearity is important for obtaining composites with high compressive strengths. On the other hand, in experiments in which the fibres were deliberately kinked [7], it was shown that the strength was approximately given by the expression

$$\sigma_{1cu} = \sigma_0 + bR, \tag{4}$$

where σ_0 and b are constants and R is the minimum radius of curvature of the fibres in the region where they were kinked. For R equal to about 5 mm the composite had the same strength as when no kink was deliberately introduced.

It is very probable that composites normally contain fibres which have significant curvature. Thus it is possible that the deliberate introduction of a minimum curvature of about 5 mm has no effect on composite strength because such a curvature was already present in the composite.

Additional evidence for fibre curvature in pultruded rods which have not been deliberately kinked comes from measurements of the compressive Young's modulus. With commercial pultruded glass–epoxies this modulus can be much less than the tensile modulus (in tension, the moduli of these materials are quite close to the Rule of Mixtures values). In the work of Piggott and Harris [4], while the compressive Young's modulus of glass–polyesters was very close to the Rule of Mixtures value, with carbon–polyesters it was very much less. In addition, the modulus of the glass–polyesters was reduced dramatically by using uncured polyesters for the matrix.

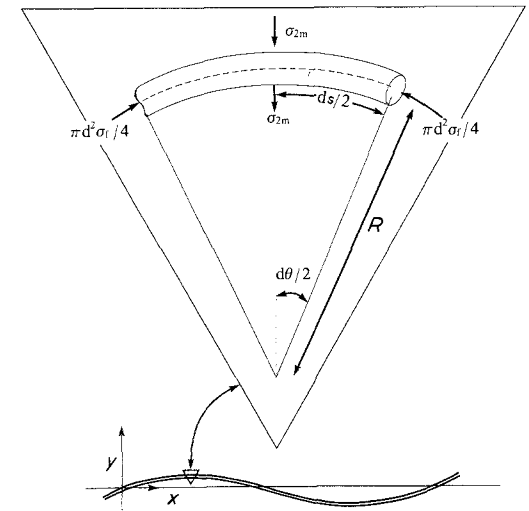

Figure 1 Fibre profile assumed with, inset, a section of fibre at an antinode. (The fibre curvature is greatly exaggerated in the inset.)

3.1. Sinusoidal fibres

A theoretical framework to describe the compressive properties of composites containing fibres which are not perfectly straight can be constructed by adapting the sinusoidal fibre model of Swift [9]. (Using a sine wave to describe the fibre profile has the advantage that any profile can be obtained by summing sines of different wavelengths.)

We will consider fibres that, initially at least, are well stuck to the matrix, so that when the composite is compressed and the fibres flex to assume a sharper curvature at the antinodes, the matrix exerts equal stresses on the inside and outside of the curve, as shown in Fig. 1.

We will use dimensionless parameters x, y, a and λ to characterize the sine curve; thus

$$y = a \sin\left(\frac{2\pi x}{\lambda}\right), \tag{5}$$

where d is the fibre diameter and yd is the actual displacement, xd is the distance along the fibre and ad and λd are the amplitude and wavelength of the sine curve, respectively.

For equilibrium of a short length of fibre, $\mathrm{d}s$, at the antinodes

$$2d\sigma_{2m}\mathrm{d}s = \frac{\pi d^2 \sigma_f \mathrm{d}\theta}{4}, \tag{6}$$

where σ_{2m} is the transverse stress exerted by the fibre on the matrix in the y-direction. Re-arranging Equation 6 gives

$$\sigma_f = \frac{8}{\pi d} \cdot \frac{\mathrm{d}s}{\mathrm{d}\theta}\sigma_{2m}. \tag{7}$$

Now $\mathrm{d}s/\mathrm{d}\theta$ is the radius of curvature of the fibres, R, and R can be obtained by differentiating Equation 5. We are interested in the region where the stresses are highest, i.e. where R is a minimum. This is at the antinode, where $\mathrm{d}y/\mathrm{d}x = 0$, and $d/R = \mathrm{d}^2y/\mathrm{d}x^2$. Thus the minimum value of R is

$$R = \frac{d\lambda^2}{4\pi^2 a} \tag{8}$$

and substituting this into Equation 7 gives

$$\sigma_f = \frac{2\lambda^2}{\pi^3 a}\sigma_{2m}. \tag{9}$$

As the composite stress is increased σ_f will increase and thus so will σ_{2m}. Unless some other failure process intervenes (e.g. fibre yielding and failure) σ_{2m} will eventually become so large that the fibre separates from the matrix on the inside of the curve, or the matrix yields, so that the fibre can push it aside. In either case, σ_f reaches some maximum value, σ_{fmax}, and the composite fails at a stress given by Equation 3. With reinforced plastics, the composite failure strain is usually less than the matrix yield strain in compression [4], so we can use Equation 2 as well, and write

$$\sigma_{1cu} = \sigma_{fmax}(V_f + V_m E_m/E_f). \tag{10}$$

Another consequence of curved fibres is a reduction in composite modulus. Straight fibres will decrease in length when compressed, with a compliance of $1/E_f$. Curved fibres will suffer an increase in a and a decrease in λ as a result of pushing against the matrix at right angles to the fibre alignment direction. This will contribute an additional compliance $1/E_{fl}$ which, for long specimens, with a length greater than λ, is given by

$$E_{fl} = \frac{\lambda^4 E_m}{\pi^5 a^3}. \tag{11}$$

(Note that we have dropped the 2 appearing in the denominator in Swift's equation because we are assuming, for the moment, that fibres and matrix adhere perfectly.) The composite modulus is

$$E_1 = V_f \Big/ \left(\frac{1}{E_f} + \frac{1}{E_{fl}}\right) + V_m E_m. \tag{12}$$

We will now discuss some experimental results that can be explained with the help of this analysis.

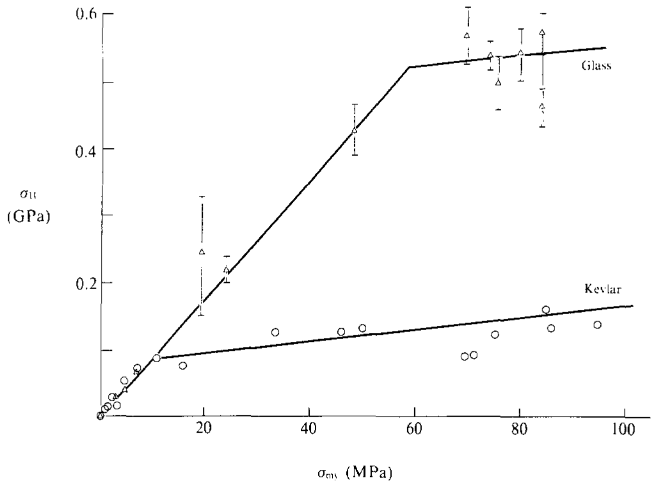

Figure 2 Composite strength against polyester matrix yield strength. (For clarity the error bars [4] have been omitted for the glass composites with $\sigma_{my} < 10$ MPa. For the Kevlar composites the standard deviations are normally less than the radii of the circles indicating the results.)

3.2. Soft matrices

If we assume perfect adhesion between fibres and matrix, σ_f should reach σ_{fmax} when $\sigma_{2m} = \sigma_{my}$. In this case Equation 9 gives us a value for σ_{fmax} which can be inserted in Equation 10 to give

$$\sigma_{lcu} = \frac{2\lambda^2}{\pi^3 a}\left(V_f + \frac{V_m E_m}{E_f}\right)\sigma_{my}. \qquad (13)$$

Then, as long as we can neglect E_m/E_f, we find that for geometrically similar composites $\sigma_{lcu} \propto \sigma_{my}$. This was indeed the case with glass-reinforced soft polyesters for σ_{my} ranging from about 0.3 to 60 MPa for $V_f = 0.31$. The equation

$$\sigma_{lcu} = 9\sigma_{my} \qquad (14)$$

describes the results over this very large range of values of σ_{my} with great fidelity.

Above 60 MPa the strength does not change significantly and it is therefore very likely that a difference failure process takes over.

To test this hypothesis, Kevlar composites were made by the same process, with $V_f = 0.31$. With Kevlar, as indicated in Section 2, we expect failure to be controlled by fibre compressive failure. However, if we go to soft enough matrices, we would expect the fibres to be able to push the matrix aside, for all values of $\sigma_{fmax} < \sigma_{fcu}$. Such indeed was the case. Fig. 2 shows the plot obtained with Kevlar (with the glass results also). The Kevlar results fit Equation 14 with σ_{my} in the range 0.3 to 3 MPa and Equations 1 and 2 with σ_{my} in the range 10 to 80 MPa.

These composites should also obey Equations 11 and 12 for their moduli. This was the case, too, unless the material was near, or above the transition between (a) matrix yielding controlled failure and (b) failure by other processes. Fig. 3 shows a plot of E_{fl} against σ_{my}. The line is drawn for $E_{fl} = 160\,E_m$ ($E_m = 43\,\sigma_{my}$ for these polyesters [4]). Most of the results below the transition are on, or very close to, the line.

For values of σ_{my} from a little below the transition to the maximum value tested, E_{fl} falls to a new line at about $30\,E_m$ for Kevlar. With glass it falls to about $80\,E_m$, though the decrease does not occur until a little above the transition. Fig. 4 shows a linear plot of Equation 12, using $E_{fl} = 160\,E_m$. It fits the glass–polyester results much better than Piggott and Harris's original inverse exponential expression [4].

From the modulus result $E_{fl} = 160\,E_m$, and the strength result $\sigma_{lcu} = 9\,\sigma_{my}$, values of a, λ and R can be calculated. The results are, in fibre diameters, $a = 4$, $\lambda = 43$ and $R = 11$. These values can only be reconciled with other observations [7] if the value used for d is many fibre diameters. Thus it was observed that a deliberately introduced radius of curvature of 5 mm had no effect, while more sharp curvatures (smaller radii) decreased the strength. If we assume a curvature equivalent to 5 mm is present, this means that $d = 5/11$ mm, so that 2100 fibres have to be acting together to give the required d. (Note that in the samples which had no deliberately introduced curvature, some of the curvature could have come from the resin cure shrinkage: if the whole fibre length is in the form of a sine curve with $a = 4$ and $\lambda = 43$, the distance over which it extends is reduced by about 7% compared with the straight fibre.)

The stress–strain curves obtained with soft matrices strongly indicate a ductile type of failure. Thus, completely rounded stress–strain curves

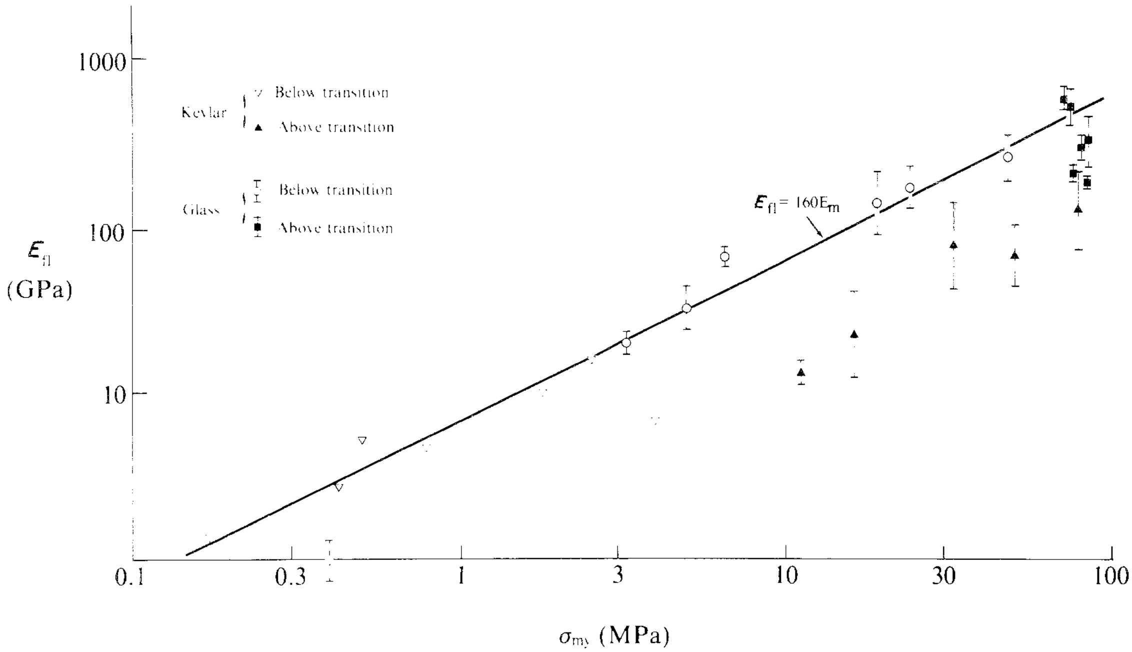

Figure 3 E_{fl} for Kevlar and glass–polyesters for a range of matrix yield stresses above and below the composite strength transitions.

were obtained with the softest matrices and rounded multiple peaks were observed with the moderately soft polymers [4]. After the stress had reached its maximum value it fell slowly and kinks could be seen to be developing in the specimen.

3.3. Hard matrices

The fully cured polyester and epoxy composites failed by splitting, rather than the controlled fibre kinking described in the previous section, though some kinking accompanied the splitting [4].

Even when the fibres are straight, transverse tensile stresses are present. These are much too small to cause splitting, however; a recent estimate

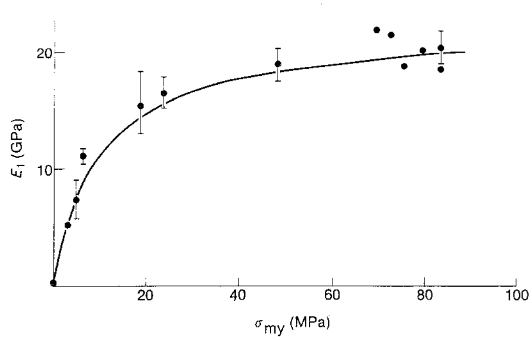

Figure 4 Composite modulus against matrix yield stress for glass–polyesters [4]. The curve shown is drawn for $E_{fl} = 160\,E_m$. (The absence of an error bar indicates a very small standard deviation.)

[10] gives the maximum stress at the fibre–matrix interface as

$$\sigma_r = \sigma_{lm}(\nu_m - \nu_f)(0.48 + 0.52V_f - 0.12V_f^2), \tag{15}$$

where ν_m and ν_f are the Poisson's ratios of matrix and fibres. For an aligned glass–polyester failing in compression with $\sigma_{fmax} = 1.3$ GPa, σ_r comes to only 5.2 MPa. With carbon, having ν_f almost equal to ν_m, σ_r is very small indeed.

However, large tensile stress can be introduced by curved fibres if many fibres co-operate, and this can lead to failure of the bond, especially when fibre–matrix bonding is poor. Three distinct strengths are involved in splitting behaviour, as shown in Fig. 5. In addition to the adhesive strength, σ_a, the cohesive strength of the matrix, σ_{mtu}, and the compressive strength, σ_{mcu} (or the yield strength, σ_{my}) will be involved.

A number of possible failure modes may be identified. We have already discussed, in the previous section, the good adhesion case, in which we have σ_{my} operating on both sides of the composite. These soft matrices do not appear to have an ultimate compressive strength (see Fig. 13 in Piggott and Harris [4]) so it was appropriate to use σ_{my}. However, the harder matrices do have an ultimate compressive strength, and we should write, instead of Equation 13

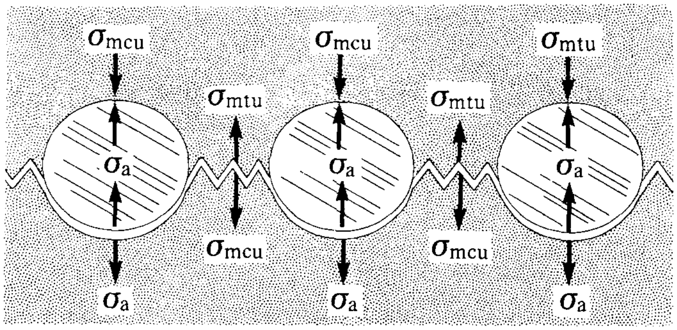

Figure 5 Stresses involved in the splitting failure of a composite.

$$\sigma_{lcu} = \frac{8R}{\pi d}\left(V_f + \frac{V_m E_m}{E_f}\right)\sigma_{mcu}. \qquad (16)$$

(Here we have replaced $2\lambda/\pi^2 a$ by $8R/d$, as well as using σ_{mcu} instead of σ_{my}: in our future discussion we shall use R/d instead of $\lambda^2/4\pi^2 a$.)

If debonding occurs, we have to overcome the matrix cohesive strength in the webs of matrix between the fibres. Allowing for the relative amounts of area over which each stress operates, we find that the composite strength is

$$\sigma_{lcu} = \frac{4R}{\pi d}\left\{\pi\sigma_a + \left[\sqrt{\left(\frac{P_f}{V_f}\right)^{1/2}} - 2\right]\sigma_{mtu}\right. \\ \left. \times \left(V_f + \frac{V_m E_m}{E_f}\right)\right. . \qquad (17)$$

(Here we have assumed that once debonding and cracking has taken place, the composite is weakened. We also assume that σ_a is the same in all directions but operates over the whole area, $\pi d/2$, rather than the resolved area, d, per unit length. P_f is the packing factor [11], equal to $2\pi/3^{1/2}$ for hexagonal packing and π for square packing.)

The results of Martinez *et al.* [7] gave a linear relation between σ_{lcu} and R, as expected from Equation 17. However, the σ_{lcu} against R plot has an intercept, σ_0, on the strength axis (Equation 4).

A positive intercept would be expected, since the composite will have some residual strength even when the fibres are extremely badly buckled. However, the intercept is rather large (~0.5 GPa for the good adhesion cases) and thus some other mechanism must be at work. For example, there will be residual stresses present due to matrix cure shrinkage.

Debonding will also reduce E_{fl} to about half the value given by Equation 11. This probably accounts for the low values of E_{fl} for glass–polyesters with $\sigma_{my} > 70$ MPa (they are about one-half the values given by $E_{fl} = 160\,E_m$, Fig. 3).

3.4. Adhesion and volume fraction

We expect from Equation 17 that the slope of the σ_{lcu} against R plot will be equal to $4\{\pi\sigma_a + [(P_f/V_f)^{1/2} - 2]\sigma_{mtu}\}(V_f + V_m E_m/E_f)/\pi d$. Thus it should be different for different fibre volume fractions and different levels of fibre–matrix adhesion. This is indeed the case. For perfect adhesion, we replace $\pi\sigma_a$ by $2\sigma_{mtu}$ (it is expected that the matrix, rather than the bond, fails under the fibre in this case). We then find that for glass–polyester the slope for $V_f = 0.50$ should be 1.23 times that for $V_f = 0.30$. In the experiment [7] this ratio was 1.5 ± 0.4 when the adhesion was good. When adhesion was severely reduced by removing the silane coating from the glass the ratio fell to 0.28 ± 0.14. For very small σ_a the theoretical ratio is much larger than this, i.e. 0.74. Thus, while Equation 17 predicts the trends correctly the actual values are not so well predicted. This could well be because of extraneous factors, such as poor wet-out of the high V_f poor adhesion samples, leading to very low effective values of σ_{mtu}.

The combined effects of adhesion and fibre volume fraction are illustrated in Fig. 6. The curves for splitting failure were plotted using Equation 17 and 50% adhesion means that $\sigma_a/\sigma_{mtu} = 0.5$. The lines for transverse compression failure were plotted using Equation 16.

Strongly non-linear V_f effects are predicted in this plot. For example, consider a polymer matrix which is 50% stronger in compression than in tension, i.e. $\sigma_{mcu} = 1.5\,\sigma_{mtu}$. It is expected to fail by "transverse compression" (i.e. the sideways push of the fibres will exceed σ_{mcu}) with V_f in the range 0 to 0.4 if the adhesion between fibres and matrix is perfect. Above $V_f = 0.4$ splitting failure is predicted. If the adhesion is quite poor, e.g. $\sigma_a = \sigma_{mtu}/4$, splitting failure is predicted to start

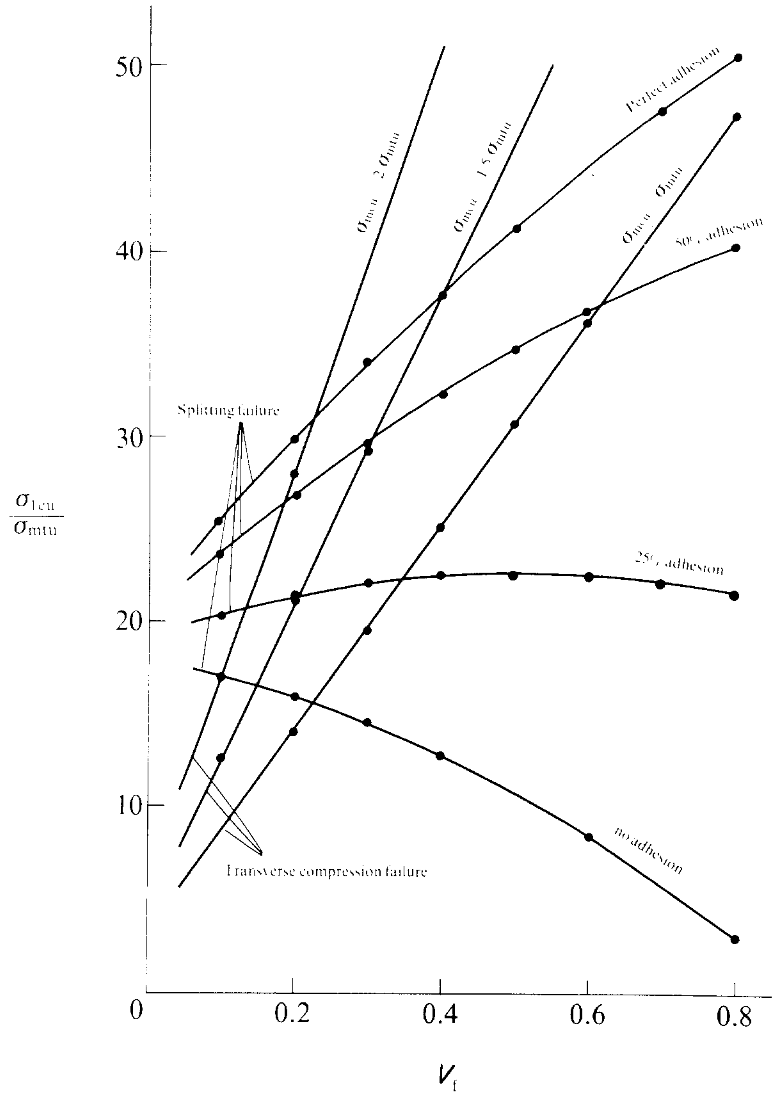

Figure 6 Dimensionless plot for composite compressive strength when controlled by transverse splitting and transverse matrix compression failure.

at $V_f = 0.2$. In both cases the strength against V_f plot starts off linearly and then falls below the line at higher fibre volume fractions. This type of plot with $\sigma_{mcu} \simeq 1.5\,\sigma_{mtu}$ can be made to fit the results of Martinez *et al.* [7] quite well. The more gradual effects observed by Hancox [12] with carbon–epoxies cannot easily be explained by this combination of failure processes, however.

3.5. Fibre–fibre interactions

In our discussion of effects due to fibre curvature, we have already concluded that fibre interactions take place, so that the effective fibre diameter is much larger than individual fibre diameters. With hybrid composites, "hybrid effects" have been observed in both compressive strengths and compressive Young's moduli [13]. These "hybrid effects" indicate fibre interactions. However, in this section we will first discuss systems in which interactions appear not to occur.

When Kevlar was combined with glass or carbon the composite strength was given by the rule of mixture expression (i.e. there was no "hybrid effect"):

$$\sigma_{1cu} = V_{fk}\sigma_{fku} + V_{fg}\sigma_{fmax} + V_m\sigma_{ml}, \quad (18)$$

where the subscript "k" stands for Kevlar and the "g" stands for glass (or carbon). Since Kevlar is a ductile fibre, this type of behaviour is to be expected. As the composite strain is increased from zero in the compression test, the first event that occurs is the yielding of the Kevlar, when the strain is 0.4 to 0.5%. At higher strains (up about 5%) the stress in the Kevlar changes very little.

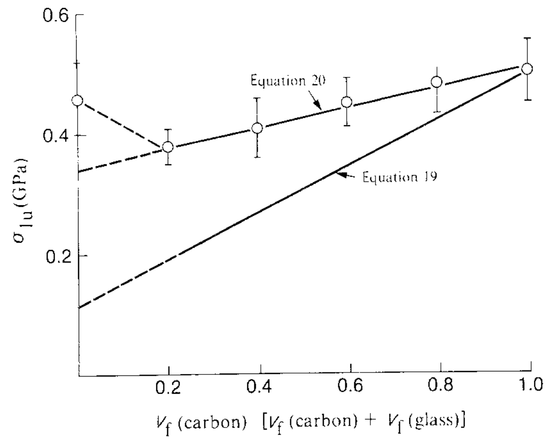

Figure 7 Strength of hybrid composites [13].

Thus the Kevlar is taking approximately the same stress, σ_{fku}, while the stress in the carbon or glass, σ_f, is increasing. When σ_f reaches σ_{fmax}, the composite fails (note that at this stage the Kevlar is providing little or no lateral support). Hence each fibre makes the strength contribution indicated in Equation 18.

The glass–carbon hybrids show negative "hybrid effects" for compression strength. In Fig. 7 the results for glass–Kevlar and glass–carbon are compared.

With brittle fibre combinations, where the fibre Young's moduli differ considerably, a negative "hybrid effect" can be expected because the high modulus fibre reaches σ_{fmax} before the low modulus one does. Using the subscripts "hi" for the high modulus fibre and "lo" for the low modulus one, we might expect failure to occur when $\sigma_{fhi} = \sigma_{fmax}$. At this strain, the stress in the other fibre is $\sigma_{fmax}E_{flo}/E_{fhi}$. Thus the compression strength is

$$\sigma_{lcu} = \sigma_{fmax}\left(V_{fhi} + \frac{V_{flo}E_{flo}}{E_{fhi}} + \frac{V_m E_m}{E_{fhi}}\right). \qquad (19)$$

A plot of Equation 19 is included in Fig. 7, The "hybrid effect" predicted is clearly much too big, indicating σ_{fhi} can be greater than σ_{fmax}, probably because the lower modulus fibres can assist the matrix in resisting the push of the higher modulus fibres. An expression which fits the results shown in Fig. 7. is

$$\sigma_{lcu} = \sigma_{fmax}\left(V_{fhi} + \frac{3V_{flo}E_{flo}}{E_{fhi}} + \frac{3V_m E_m}{E_{fhi}}\right) \qquad (20)$$

where the multiplier "3" is introduced to account for the supporting effect of the lower modulus fibres, once the higher modulus fibres reach σ_{fmax}.

These mechanisms cannot explain the positive "hybrid effect" observed with the high modulus, high strength all-carbon hybrids, or the negative "hybrid effects" observed with the moduli of all types of hybrid tested by Piggot and Harris [13].

4. Conclusions

Relatively simple ideas can be used to explain the compressive strengths and moduli of fibre composites. When the fibres are ductile, the strength of the composite is usually dominated by the fibre strength. When the matrix is very soft it controls the compressive strength and modulus, probably as a result of built-in fibre curvature that causes lateral stresses. Qualitative agreement with experimental results is very good, but quantitative agreement depends on making some allowance for fibre–fibre interactions. These interactions are particularly important in hybrid composites.

To make composites with high compressive strengths and moduli the fibres should be hard, to avoid ductile failure and as straight as possible to keep transverse stresses as low as possible. To inhibit splitting failure the fibre–matrix bond should be good and the matrix should have high tensile strength. To prevent matrix transverse failure the matrix should have high compressive strength and yield strength.

Hybridization can be used to improve the compressive strength of composites containing ductile fibres such as Kevlar, but should be avoided with brittle fibres because of unfavourable "hybrid effects".

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